Faraday Rotation in Air: Combining Brute Force and Finesse

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Abstract

Because of the weak circular birefringent response to axial magnetic fields, and hence low values of the Verdet constants, only a few research groups have successfully measured non-resonant Faraday Rotation in diamagnetic gases. In contrast to those complicated schemes, we utilized a straightforward technique resulting in accurate measurements of the Faraday rotation in air. Our method combines brute force (a magnetic field intensity of around 210 gauss over a 60 cm length), and finesse (differential phase sensitive detection), resulting in accurate values for the Verdet constant of air, $V_{air,}$ at a dozen wavelengths ranging from 405-800 nm, which can be modeled as a mixture as V_{air} = (0.7809 V_{N2} + 0.2095 V_{O2}).

1. Introduction

The Faraday Effect, named for physicist Michael Faraday, is a magneto-optical phenomenon characterizing the rotation of the plane of polarization of light – a form of optical activity due to Zeeman splitting.

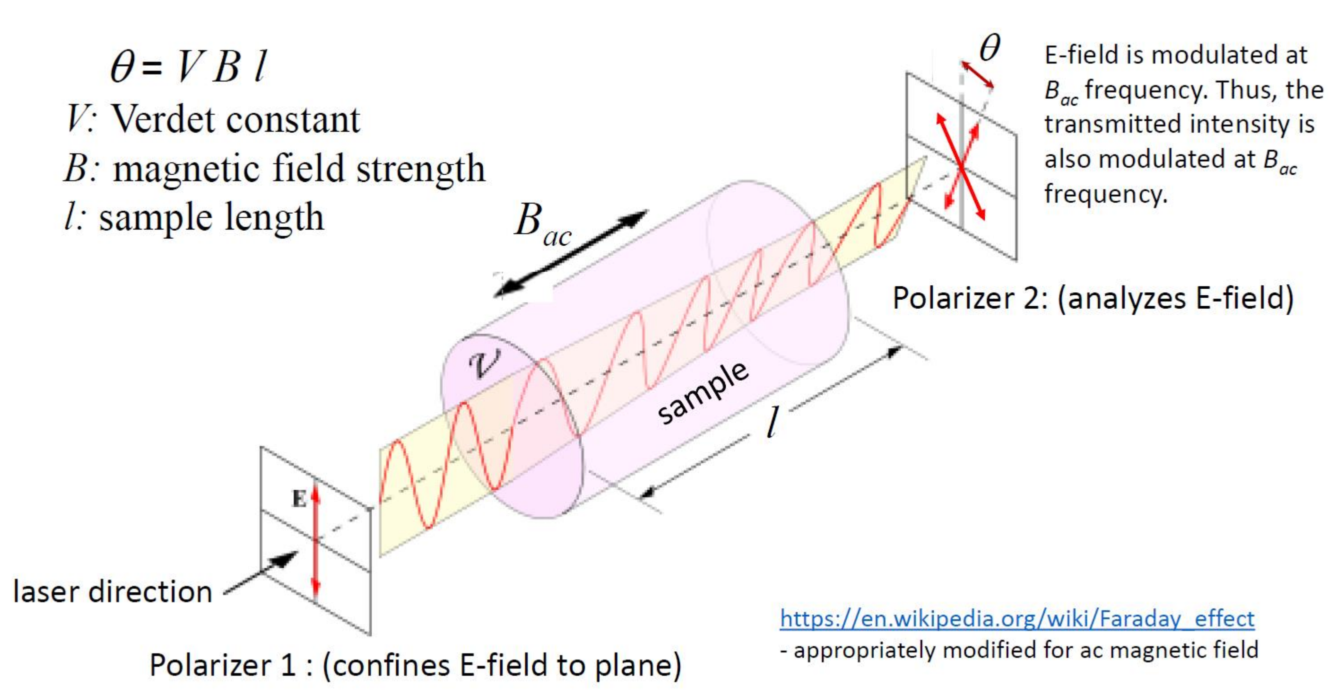


Figure 1. A longitudinal AC magnetic field in the sample (air), induces optical activity in the polarized collinear laser beam. The electric field (polarization direction) of the laser light is modulated at the same frequency as the applied magnetic field and the intensity of the light after polarizer 2 is modulated at the same frequency.

The angle of rotation, θ is dependent on the strength of the magnetic field (i.e. the magnetic flux density), B, the length, l of the sample, and the Verdet constant, V according to

$$\theta = V B l$$

The Verdet constant, also dependent on wavelength (and to a lesser extent on temperature), characterizes the Faraday effect in a transparent material.

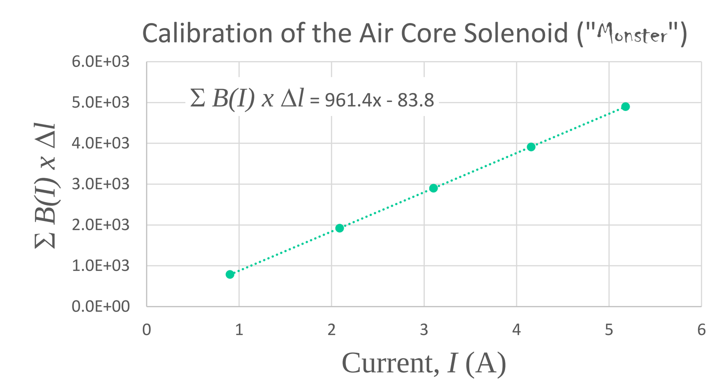
2. Experimental Method (Theory I)

Lock-in amplifiers (LIAs) record an input signal as RMS volts. The rotation, induced by a modulating magnetic field, follows from the relations[1,2]

$$\theta = \frac{1}{4}\sin^{-1}\left(\frac{V_{AC(A-B)}}{V_{DC,A}+V_{DC,B}}\right) \sim \frac{V_{AC(A-B)}}{4\left(V_{DC,A}+V_{DC,B}\right)}$$

that are valid for the small rotations occurring in our apparatus (Fig. 3). The Verdet constant is extracted using measured quantities defined by a measured differential AC voltage signal, $V_{AC(A-B)}$, respective DC voltage signals, $V_{DC,A}$ and $V_{DC,B}$; utilizing a lock-in amplifier and voltmeters. The magnetic flux density, B, is a function of the supplied current, I and is obtained via solenoid calibration (Fig. 2)

2. Experimental Method (cont.)



V_{DC}: constant photodiode signal (auxiliary input of the LIA)

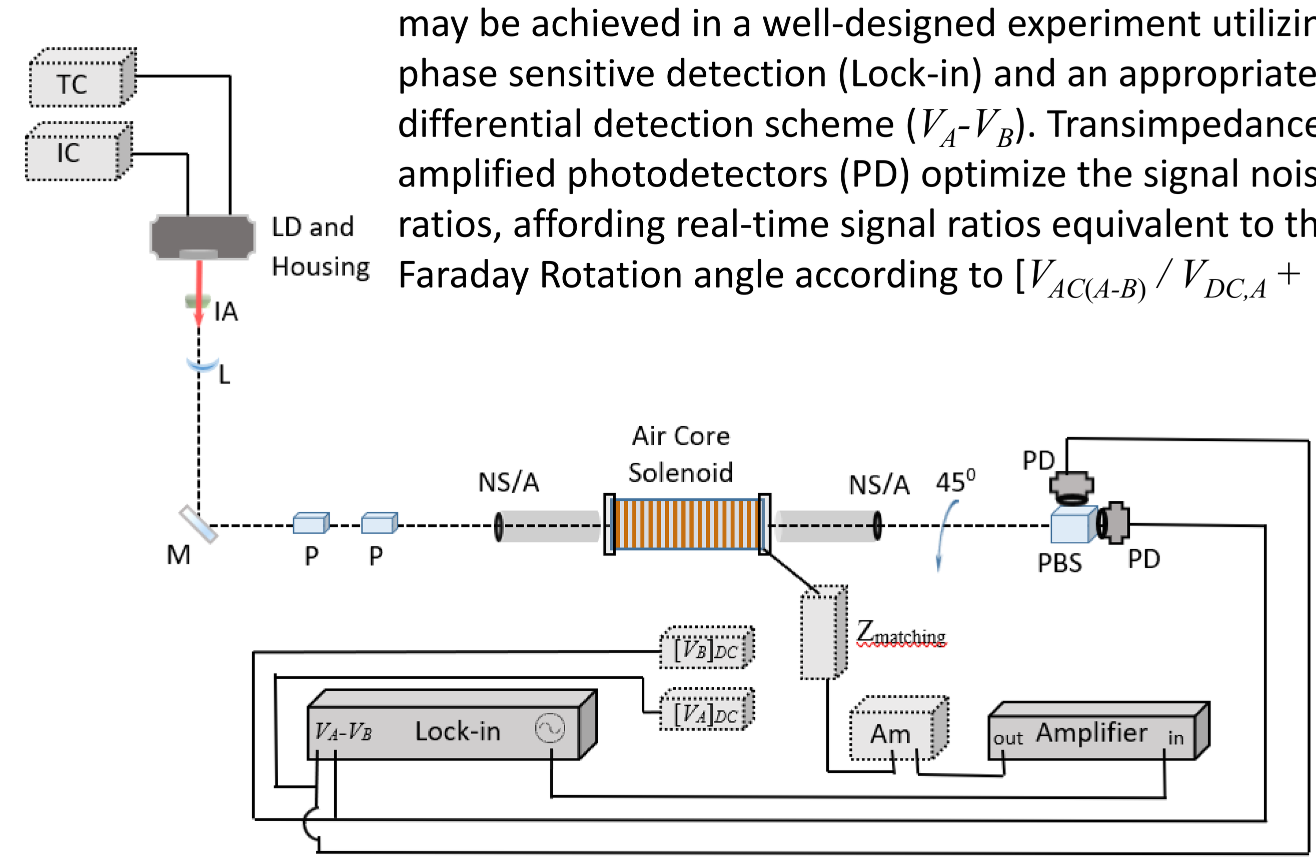
V_{AC}: referenced modulated signal (the PSD input of the LIA)

Bl: solenoid calibration (Figure 2 - left); B is applied along the sample length, l

Figure 2. Calibration of the solenoid

After calibrating the solenoid (Figure 2), the magnetic field strength is determined as a function of the monitored current, I supplied to the solenoid. The Verdet constant of the sample follows from $V = \theta / Bl$.

3. Apparatus



Experimental Technique: Signal to noise ratios exceeding 10^{-5} may be achieved in a well-designed experiment utilizing phase sensitive detection (Lock-in) and an appropriate differential detection scheme (V_A-V_B). Transimpedance-amplified photodetectors (PD) optimize the signal noise ratios, affording real-time signal ratios equivalent to the Faraday Rotation angle according to [$V_{AC(A-B)}$ / $V_{DC,A}$ + $V_{DC,B}$].

Experimental Arrangement: LD-laser diode , TC, IC- temperature and current controllers,* IA- intensity attenuator, L- convex lens, M-turning mirror, P-crystal polarizer, NS/A-noise suppressing tube (B-Field) with aperture, PBS- polarizing beam splitter, PD-photodetector, Am-ammeter, $[V_{A,B}]_{DC}$ - voltmeters for V_A,V_B averaging DC, Z_{match} - resistor bank (~4Ω) impedance for audio amplifier

Lock-in amplifier: SRS830-LIA [inputs: differential A-B {V_A-V_B }

V_{AC} (PSD sig.), outputs: AC voltage & ref freq, 2 Voltmeters $[V_{A,B}]_{DC}$ (Siglent SDM3045x) for average DC signals

Determining the induced rotation (Signal outputs): by averaging simultaneous snapshots (smartphone) of Lock-in V_A-V_B, both Voltmeters $[V_{A,B}]_{DC}$, and Am-Ammeter (VICI VC8145) for measuring solenoid current.

*The laser diode, TC, and IC controller are interchangeable devices.

4. Data Analysis (Theory II)

The major constituents of air (to 99.04%) are nitrogen (78.09%) and oxygen (20.95%). Previous researchers[3] have experimentally determined the Verdet constants (normalized to atmospheric pressure) of those gases using wavelengths of 400-700nm. Using that data, we tabulated the Verdet constants of a weighted N2 + O2 mixture (dispersion curves, Figs 4,5). For comparison, our measurements in air are graphed along with those results. We show, as expected, that the Verdet constant of air, represented by our measurements, can be quite accurately calculated from the Verdet constants of the appropriately weighted values of an N2 + O2 mixture.

5. Results

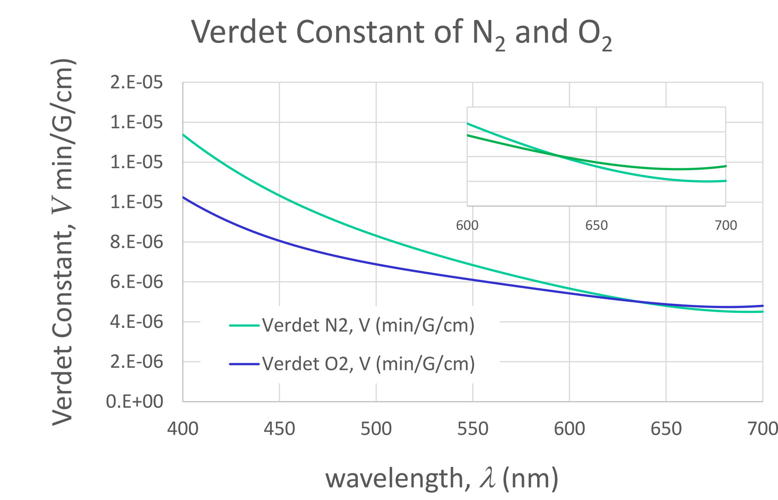


Fig. 4 (left): The graph shows 4[th] order polynomial fits intended only to provide a trendline to the experimental data of the Verdet constants for N_2 and O_2 (refs 3,4). It is evident that measurements in air in the region around the crossing near 635nm are most likely to agree with weighted sums Fig.5 below.

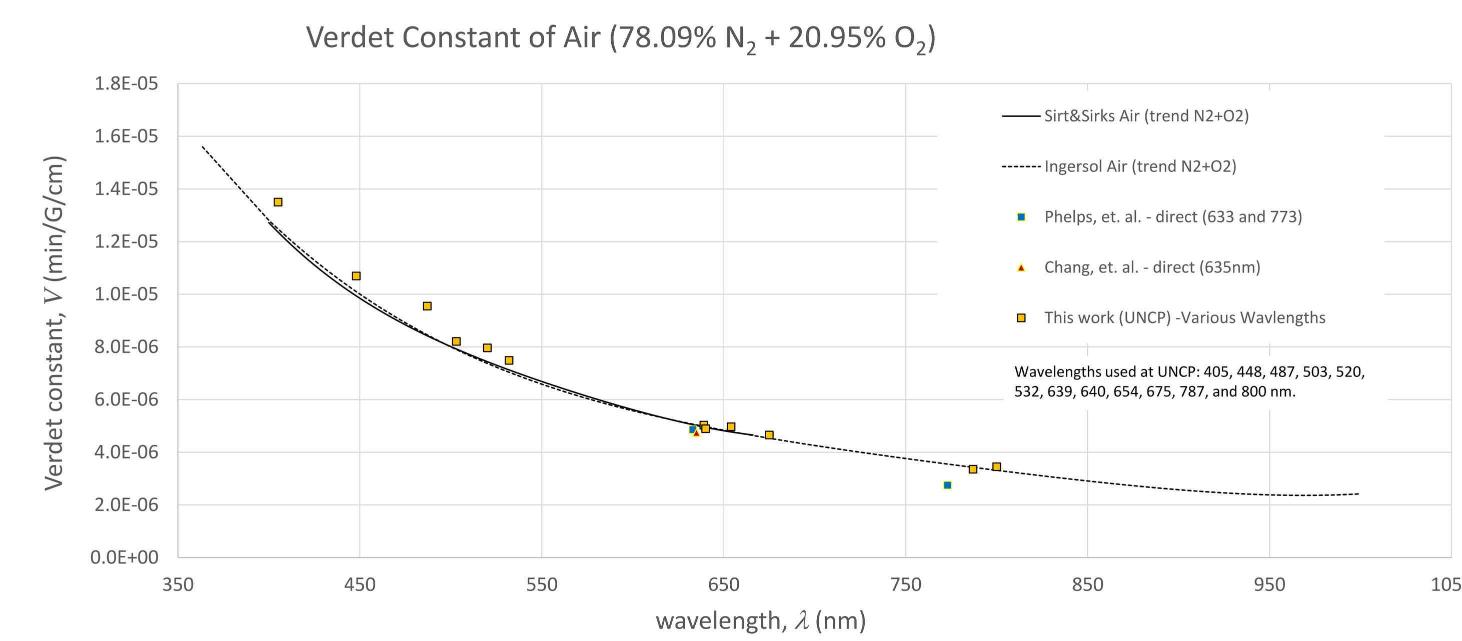


Fig. 5 (above). Verdet constant , V, of air based on all known measurements in the visible and near infrared wavelength range. The two trendlines were generated by normalizing appropriately weighted 4[th] order polynomial fits to measurements of N_2 and O_2 (ref 3). Our measurements (gold squares with black outlines) are in excellent agreement with the other three previous measurements, in addition to the assumption that V_{air} = (0.7809 V_{N2} + 0.2095 V_{O2}).

6. Conclusion

Exploiting a strong magnetic field (**brute force**) over a lengthy solenoid in conjunction with the sensitivity of a lock-in amplifier configured in a differential mode (**finesse**), we were able to accurately measure the dispersion of Verdet constant of air in a relatively simple experimental arrangement. Reduced signal noise from the small laser intensity fluctuations utilizing differential detection were required to enhance the phase sensitive detection capabilities of our lock-in amplifier, overcoming S/N ratios near 10^{-6}, an order of magnitude improvement over the often-quoted S/N ratio of 10^{-5}. Going forward, we look to explore additional configurations that could further increase accuracy and overcome even lower S/N ratios, such as double modulation and auto-balanced photodetection. In the former scheme, two lock-in amplifiers, one phased locked to an amplitude modulated laser, and the other phase locked to the ac magnetic field is proposed. In the latter scheme, a Nirvana 2007 auto-balanced optical receiver, which is very effective at eliminating laser intensity noise (i.e., shot-noise limited measurements) in conjunction with a single lock-in amplifier phase locked to an amplitude modulated laser could also prove very effective.

6. References

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